$(a_1\ a_2\ \dots\ a_n) = (a_n\ \dots\ a_2\ a_1)$. Hence proved.

2.1.12. Even and Odd Permutation

Even permutation are those permutation which the number of transposition in a cycle permutation is even, while it is known as **odd permutation** in case it has odd number of transposition in a cycle permutation.

For example, If $f = \begin{pmatrix} 1 & 4 & 7 & 8 & 5 & 3 & 6 & 9 \\ 4 & 1 & 9 & 6 & 3 & 5 & 8 & 7 \end{pmatrix}$ then it is an even permutation as f is the product of four (even) transposition (1 4)(7 9)(6 8) and (3 5).

Some Important Results

1) A permutation can-not be odd and even at the same time.

2) The product of (n–1) transpositions can be used to represent a cycle of length n. Consequently, it should be noted that if n is odd, a cycle of length n will be an even permutation; if n is even, it will be an odd permutation.
3) Every transposition is an odd permutation.
4) Identity permutation is always an even permutation.
5) The product of two even permutations is an even permutation.
6) The product of two odd permutations is an even permutation.
7) An odd permutation is produced when an even permutation and an odd permutation are combined, and vice versa.
8) An odd permutation has an odd inverse, while an even permutation has an even inverse.

Theorem 7: A permutation cannot be both even and odd, i.e., if a permutation f is expressed as a product of transpositions then the number of transpositions is either always even or always odd.

Proof: Let us consider the polynomial P in n distinct symbols $x_1, x_2, \ldots, x_n$ which is defined as the product of all factors of the type $(x_i - x_j)$, where $i < j$.

Therefore, $P = \prod_{i<j=1}^{n} (x_i - x_j)$

$= (x_1 - x_2)(x_1 - x_3)(x_1 - x_4) \ldots (x_1 - x_n)(x_2 - x_3)(x_2 - x_4) \ldots (x_2 - x_n)$

$\ldots\ldots(x_{n-2} - x_{n-1})(x_{n-2} - x_n)(x_{n-1} - x_n)$.

Suppose f is any permutation on n symbols 1, 2, 3, ..., n.

Now f P is the polynomial obtained by changing subscripts of a' s in P by f.

For example, if n = 4, we have
$P = (x_1 - x_2)(x_1 - x_3)(x_1 - x_4)(x_2 - x_3)(x_2 - x_4)(x_3 - x_4)$

If f = (1 2 3) then $f P = (x_2 - x_3)(x_2 - x_1)(x_2 - x_4)(x_3 - x_1)(x_3 - x_4)(x_1 - x_4)$

Particularly, if f = (2 3) (the transposition) then
$f P = (x_1 - x_3)(x_1 - x_2)(x_1 - x_4)(x_3 - x_2)(x_3 - x_4)(x_2 - x_4) = -P$.

i.e., the effect of transposition f is to change the sign of P.

If f = (r s) the transposition
Where r < s then effect of f on P is as follows:
1) Any factor of P that has neither the suffix r nor s remains unchanged.
2) On replacing r by s the single factor $(x_r - x_s)$ changes its sign and *vice versa*.
3) The remaining factors that have either the suffix r or s but not both can be grouped with a combination of the following three categories:
   i) $[(x_1 - x_r)(x_1 - x_s)]\,[(x_2 - x_r)(x_2 - x_s)] \ldots [(x_{r-1} - x_r)(x_{r-1} - x_s)]$